Journal of Applied Research in Science and Humanities

The role of adding carbon nanoparticles to free convective boundary layer viscoelastic fluid flow past a vertical porous cone surface: application in wastewater treatment

Asmaa Mohamed Abdelqwi, Bassant Mostafa Kamal, Eriny Beshay Ayad, Habiba Ahmed Salama, Karen Mohamed Abdelhakim, Toqa Hassan Mohamed, Yasmine Mohamed Atallah

Supervisor: Nasser S. Elgazery, Professor of Applied Mathematics (Fluid Dynamics)

Ain Shams University, Faculty of Education, Program General Mathematics

Abstract

In this search, we studied how to purify wastewater (agricultural water) using nanotechnology. As it offers affordable, eco-friendly solutions for wastewater treatment. The proposed physical model is expressed by studying the role of adding nanocarbon particles to the free convection boundary layer viscoelastic fluid flow past a vertical porous cone surface. The system of nonlinear partial differential equations governing the problem has been transformed by a similar transformation into a system of ordinary nonlinear differential equations, which is solved numerically by applying the Runge-Kutta technique with the shooting procedure. Graphical results for the velocity and temperature for various parametric conditions are presented and discussed. The most significant findings in this work are that the addition of carbon nanoparticles will increase water velocity, then reduce the mass of the deposit in treated water; the addition of hybrid carbon nanoparticles will decrease the mass of the sediment in agricultural wastewater treatment; and the increasing in the volume of carbon nanoparticles will increase water velocity.

Key Words: Carbon nanoparticles, Wastewater, Free convective fluid flow.

1. Introduction

The strategy of water filtration with developed even wetlands was begun in Germany by Käthe Seidel and the English social reformer and open well-being dissident Sir Edwin Chadwick in 1960. In the center of the 19th century. In these frameworks, wastewater is presented at the gulf and streams gradually through a layer of permeable media underneath the surface in a more or less flat way until it comes to the outlet where it is collected sometime recently leaving through an outlet-level control gadget. Amid this course, wastewater is uncovered in a range of high-impact, anaerobic, and anaerobic zones. There are numerous sorts of wastewater, such as household wastewater, mechanical wastewater, and agrarian wastewater (Vymazal, 2009).

Stages of water filtration; Pre-treatment is to begin with the arrangement of wastewater treatment and is utilized to prepare the water for treatment in consequent stages. So, it comprises evacuating things that can harm the plant or hardware amid the cleaning. Coarse sifting is more often than not done, to begin with. In this handle, expansive and medium-strong squanders are isolated utilizing sifters and strainers of distinctive thicknesses. After that, the oil and sand particles are evacuated with cleaning and degreasing specialists; see Fig. 2(a). The reason for this step is to expel a few lighten. For this, water is kept in decanter centrifuges for 1-2 hours, where gravity makes a difference to isolate these particles. Other points of interest of this handle incorporate stream homogenization and expulsion of natural matter related to suspended solids. Chemicals such as coagulants and floaters may moreover be included amid this handle to move forward the settling of solids and evacuate phosphorus. In a few cases, bases and acids are utilized to neutralize the pH of the water; see Fig. 2(b). The reason for the moment arranging chemical treatment is to make strides in the last quality of the water so that it can be returned to the environment (oceans, streams, lakes, and other bodies of water) and in a few cases utilized for human exercises. To accomplish this, an arrangement of forms is carried out to dispose of pathogenic substances, such as microbes. Strategies utilized incorporate filtration with layers of sand or other materials and sanitization with either chlorine (more often than not sodium hypochlorite) or UV light for decrease. the number of infinitesimal living living beings delivered in the past stages; see Fig. 2(c). In the third arrangement, this preparation is outlined to evacuate natural matter and supplements such as nitrogen and phosphorus from the water. This third organ, which is primarily organic, as a rule employments microscopic organisms and microorganisms to break down and evacuate natural matter and different supplements from the water. The most common treatment is enacted slime, where the treated water is cleared out in a tank for a few days beneath diverse oxygen conditions (high–impact, anaerobic, and anaerobic) depending on the transfer prerequisites. Here, distinctive sorts of microbes living in a tank or reactor eat the natural matter and supplements in the water, expelling them from the water and exchanging them for their life forms. A third deposition process is common after the biological process. The bacteria grown in the previous process settle to the bottom, forming a mixture of water and solids called biological sludge. This mixture is drawn off or washed through the bottom of the carafe and the purified water flows through the top, free of most bacteria and solids, creating clarified water. In water treatment plants, it is common that water treatment ends at that stage when the treated water meets the specified discharge requirements and there are no other water quality requirements for reuse or further use; see Fig. 2(d).


(a)


(b)


(c)


(d)

Figure 1. Stages of water purification.

Some researchers are making great efforts to study the mathematical model of sewage. For example, George Chopanoglos is a distinguished civil and environmental engineer who has made significant contributions to the field of water treatment technology. Richard Stoetz is a leading researcher in sewage treatment modeling with a focus on odors and mutated organic compounds. He has published many research papers in this field and has greatly advanced the understanding and control of odors in water treatment processes. Jiri Vanner is a mathematician and chemical engineer known for his work modeling wastewater treatment, particularly activated clay systems. He made a significant contribution to this field through his research and publications. Mark Van Loosdrecht is a distinguished environmental engineer and professor known for his work in biological wastewater treatment. His research focuses on mathematical modeling of systems such as activated clay, and dirt and he has made significant contributions in this field(Basso, 2023).

The prefix "dwarf" refers to the Greek prefix meaning "dwarf" or something very small and describes one thousand millionth of a meter (10^{-9} m). A nanometer (NM) is a unit of measurement equivalent to one billionth of a meter. Nanoscience studies structures and molecules at the nanometer scale of 1–100 nm. It is understood that one of the first people to use this technique was medieval glassmakers who used colloidal gold nanoparticles for painting. The idea of nanotechnology goes back to Richard Feynman, who won the Nobel Prize in physics in 1965. Here was the spark of a new science, followed by Eric Drexler, who coined the term "nanotechnology" in 1986. The nanometers are used to measure the smallest things, usually the size of an atom or molecule(Sandhu,2006). Nanomaterials can disappear in one dimension on nanoscale thin surfaces in two dimensions (nanowires–nanotubes) or all three dimensions (nanoparticles–quantum). Nanotechnology offers the opportunity to produce new nanomaterials that are used in the treatment of surface water, groundwater, and wastewater contaminated with toxic metal ions, and organic and inorganic solutions. Wastewater treatment with nanobubbles or nanofiltration systems for heavy metals are some of its environmental applications. Nanocatalysts are also available to make chemical reactions more efficient and less polluting. Applications of nanomaterials are used in the fields of electricity, biomedicine, and environment (water pollution). Using nanomaterials, we can purify water by purifying water through filtration and various nanoparticle compounds (Ag, CeO_2, graphene, CNTs) and zeolites, activated carbon, and metal–organic frameworks). The advantages of which are heat and chemical resistance, mechanical resistance, and chemical resistance(Mansoori,2005).

One of the nano types is nanocarbon. Carbon nanotube (CNT) was first discovered in 1991; see Fig. 2 and Fig. 3. Nanocarbon materials with unique properties such as large surface area, tunable surface chemistry, and exceptional adsorption capacity have emerged, e.g. promising candidates for water treatment. They offer possible solutions to various water problems. Heavy metals: Their high surface area and functional groups can capture and immobilize heavy metals such as lead, mercury, and arsenic from water(Ajayan,1999).Nanocarbons can also act as antimicrobial agents, inactivating bacteria, and viruses through physical contact or photocatalytic processes. Carbon nanotubes (CNT) their a unique structure that facilitates the passage of water and repels salts, offering the potential for energy-efficient desalination processes(Das, 2014). Nanocarbon composites with other materials, such as metal oxides, can improve their adsorption capacity and target specific targets. Wastewater impurities. certain nanocarbons, such as graphene oxide and CNTs, can be used as photocatalysts to degrade organic pollutants in wastewater when exposed to UV light. Based on the above, nanocarbon is one of the most effective species that contribute to the purification of polluted water(Thines, 2017).

Mathematicians work with scientists to develop and implement numerical methods and algorithms to simulate the properties and behavior of carbon nanostructures. For example, they can develop computer models based on quantum mechanics or molecular dynamics simulations. These simulations provide information about the structural, electronic, and mechanical properties of carbon nanostructures that complement experimental observations. Carbon nanostructures exhibit quantum mechanical effects such as electron confinement and quantum tunneling (Rubio,2013). For example, Vladimir Rokhlin introduced multi-differential homogeneous differential (MDP), a computational technique used to solve mathematical problems related to carbon nanotubes (Rokhlin,1990). Dominic Del Vecchio presented mathematical models for the analysis and design of carbon nanotubes and other nanoscale structures Thomas Tang is a mathematician and physicist who developed mathematical models for materials and for analyzing the behavior and nanotechnical properties of nanostructures(Vecchio&Giacomelli,2006).

In the second half of the 20th century, water pollution reached alarming proportions, while water consumption increased along with it. In water-scarce countries (Egypt), the rationalization and management of water consumption are combined with the improvement of agricultural water purification processes. Much work remains to be done to improve existing water treatment processes, especially to gain a better understanding of wastewater flow characteristics.

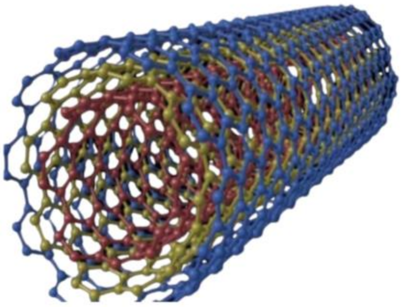

Figure 2. Carbon nanotube (MWCNT).

Figure 3. Carbon nanotube (SWCNT).

A viscoelastic fluid is a type of fluid that has both viscous (resistance to flow) and elastic (resistance to deformation) properties. The behavior of viscoelastic fluids depends on factors such as the rate of deformation, the duration of the applied force, and the internal structure of the material. Examples of viscoelastic fluids include polymer solutions, certain types of gels, and biological fluids such as blood and sewage. These materials are commonly encountered in a variety of industrial, biological, and everyday applications, including food processing, cosmetics, and biomedical engineering(Seyssiecq&Ferrasse, 2003). Viscoelastic fluids are common in critical applications. Many of them are so common that people usually do not pay attention to their importance. Paints are a very good example, as almost all buildings are painted with a combination of polymer and solvent for various reasons. More complex applications involving polymer solutions. On the other hand, not all viscoelastic fluids are paints, and some may have industrial uses, such as sewage. At the same time, some applications using viscoelastic fluids may require a physical mechanism such as natural convection to function properly(Lakes, R.S.,2009). Viscoelastic fluids are usually classified as non–Newtonian fluids, meaning that their viscosity is not constant but varies with applied stress or shear rate(Denn, 1990).

Free convection is a natural heat transfer process that occurs when a fluid is heated from below and cooled from above, creating a temperature gradient that causes the fluid to circulate. This process is caused by density differences caused by fluid temperature fluctuations and can occur in both liquids and gases.

Free convection is a common phenomenon that can be observed in many everyday situations, such as warm air rising. of cooler or cloudy formation in the atmosphere. It plays an important role in many industrial and natural processes, including heating and cooling systems, meteorology, and geophysics. In addition, free convection can improve heat transfer in many engineering applications, such as heat exchangers and solar collectors. Free convection, also known as natural convection, is the movement of fluid due to density differences due to temperature changes. This happens without an external force such as a pump. Free convection has several applications in different fields(Prep, 2023):

Heat transfer: In industrial processes, natural convection is often used for heat transfer in equipment such as heat exchangers, boilers, and coolers.

Fluid dynamics: free convection is also studied in fluid dynamics to understand the behavior of fluids under different conditions, such as in wastewater treatment.

Odor Control: Free convection can help break down and dilute foul-smelling gases from water treatment processes. The temperature differences between the agricultural wastewater and the surrounding air create buoyancy forces that promote the upward movement of air and contribute to the diffusion of odors into the atmosphere.

The essential objective of this research is to explain the role of adding nanocarbon particles to the flow of the viscoelastic fluid of the free convection boundary layer as an application for wastewater treatment and then convert the polluted water into water suitable for use in agriculture again. The system of partial differential equations is first transformed into a system of ordinary differential equation nonlinear before being solved numerically by using the 4th order Runge-Kutta method with the shooting technique. The results are then compared graphically with those of SWCNT/MWCNT to support the validity of the research(Aman, 2017).

2. Problem formulation

Consider an incompressible steady thermal laminar free convection under the Boussinesq boundary layer approximation of CNTs/H_2O viscoelastic nanofluid flowing over an isothermal vertical cone surface. The tip angle of the cone is α (constant), and for very small values of this angle, the problem under consideration can be transformed into a system of two-dimensional Cartesian coordinates (x, y). Where, x is the axis along the cone slant height, and is the axis normal to it (see Fig. 4). Also, $r(x)$ is the local radius to the point inside the boundary layer of the fluid? Since the boundary layer is very thin, $r(x)$ can be equal to its value at the cone surface. The surface of the plate maintains a uniform constant temperature T_w, which is higher than the corresponding value T_∞, and sufficiently far away from the surface. The velocity field is taken to be $(u, v, 0)$ in the direction of and respectively, and we considered that there is an injection for the fluid through the porous surface of the cone with the velocity $v = v_0(x)$, which is a function of x. Under the above assumptions, the governing equations with the boundary layer approximation are described by the following equations:

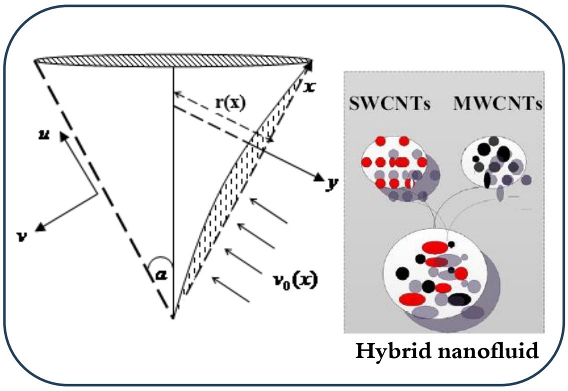

Figure 4. Schematic diagram of the mathematical modeling and geometric coordinates

The continuity equation:

$$\frac{\partial\,(r'u')}{\partial x'}+\frac{\partial\,(r'v')}{\partial y'}=0\,, \qquad (1)$$

The momentum equation:

$$\rho_{hnf}\left(u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}\right)=\mu_{hnf}\frac{\partial^2 u'}{\partial y'^2} +\lambda_0\left(\frac{\partial}{\partial x'}\left(u'\frac{\partial^2 u'}{\partial y'^2}\right)+\frac{\partial u'}{\partial y'}\frac{\partial^2 v'}{\partial y'^2}+v'\frac{\partial^3 u'}{\partial y'^3}\right) +g(\rho\beta)_{hnf}\,(T'-T'_\infty)\cos\alpha\,, \qquad (2)$$

The energy equation:

$$\left(\rho c_p\right)_{hnf}\left(u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}\right)=k_{hnf}\frac{\partial^2 T'}{\partial y^{2\,\prime}}\,. \qquad (3)$$

Where, β is the volumetric coefficient of thermal expansion, μ is the dynamic viscosity.

The system (1)–(3) with letting and for pure fluid (absent of nanofluid), has been solved by Kafoussias. He studied the problem for an ordinary Newtonian fluid. In the present work, we shall generalize with the following appropriate boundary conditions(kafoussias,1992).

$$\left.\begin{array}{llll} u'=0, & v'=v'_0(x'), & T'=T'_w\,at & y'=0 \\ u'\to 0, & \dfrac{\partial u}{\partial y}\to 0, & T'\to T'_\infty\,as & y'\to\infty \end{array}\right\}. \qquad (4)$$

Further, the thermophysical characteristics can formed as follows(Xeu,2005):

Viscosity :

$$\mu_{hnf}=D_1\,\mu_f\,;\;D_1=d_1(1-\phi_2)^{-2.5};d_1=(1-\phi_1)^{-2.5}$$

Density:

$$\rho_{hnf}=D_2\rho_f\,;D_2=\left((1-\phi_2)d_1+\left(\frac{\rho_2}{\rho_f}\right)\phi_2\right);$$

$$d_2=\left((1-\phi_1)+\left(\frac{\rho_1}{\rho_f}\right)\phi_1\right)$$

Thermal expansion:

$$(\rho\beta)_{hnf}=D_3(\rho\beta)_f\,;$$

$$D_3=\left((1-\phi_2)d_3+\left(\frac{(\rho\beta)_2}{(\rho\beta)_f}\right)\phi_2\right)$$

$$;d_3=\left((1-\phi_1)+\left(\frac{(\rho\beta)_1}{(\rho\beta)_f}\right)\phi_1\right)$$

Thermal conductivity of CNTs:

$$K_{hnf} = D_4 K_f ;$$

$$D_4 = \left(\frac{1-\phi_2 + 2\phi_2 \left[\left(\frac{K_2}{K_2 - d_4 K_f} \right) Ln \left(\frac{K_2 + d_4 K_f}{2 d_4 K_f} \right) \right]}{1-\phi_2 + 2\phi_2 \left[\left(\frac{d_4 K_f}{K_2 - d_4 K_f} \right) Ln \left(\frac{K_2 + d_4 K_f}{2 d_4 K_f} \right) \right]} \right) d_4 ;$$

$$d_4 = \left(\frac{1-\phi_1 + 2\phi_1 \left[\left(\frac{K_1}{K_1 - K_f} \right) Ln \left(\frac{K_1 + K_f}{2 K_f} \right) \right]}{1-\phi_1 + 2\phi_1 \left[\left(\frac{K_f}{K_1 - K_f} \right) Ln \left(\frac{K_1 + K_f}{2 K_f} \right) \right]} \right)$$

Specific heat:

$$(\rho c_p)_{hnf} = D_5 (\rho c_p)_f ;$$

$$D_5 = \left((1-\phi_2) d_5 + \left(\frac{(\rho c_p)_2}{(\rho c_p)_f} \right) \phi_2 \right);$$

$$d_5 = \left((1-\phi_1) + \left(\frac{(\rho c_p)_1}{(\rho c_p)_f} \right) \phi_1 \right)$$

Where 1 refers to SWCNTs and 2 refers to MWCNT

To simplify the governing equations (1)–(4), we may introduce the following the dimensionless variables:

$$\begin{aligned} & x = \frac{x'}{L}, y = \frac{y'}{L}, r(x) = \frac{r'(x')}{L}, \\ & u = \frac{L}{\nu_f} u', \ v = \frac{L}{\nu_f} v', \ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty} \end{aligned} \tag{5}$$

where L is the cone slant height, the system of equations (1)–(3) and boundary conditions (4) become:

$$\frac{\partial (r u)}{\partial x} + \frac{\partial (r v)}{\partial y} = 0 , \tag{6}$$

$$\begin{aligned} & u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{D_1}{D_2} \frac{\partial^2 u}{\partial y^2} + \frac{1}{D_2} \frac{\lambda_0}{L^2 \rho_f} \times \\ & \left(\frac{\partial}{\partial x} \left(u \frac{\partial^2 u}{\partial y^2} \right) + \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right) + \frac{D_3}{D_2} Gr \, \theta , \end{aligned} \tag{7}$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{D_4}{D_5} \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} , \tag{8}$$

$$\left. \begin{aligned} & u = 0, \quad v = v_0(x), \quad \theta = 1 \quad at \quad y = 0 \\ & u \to 0, \quad \frac{\partial u}{\partial y} \to 0, \quad \theta \to 0 \quad as \quad y \to \infty \end{aligned} \right\} . \tag{9}$$

Where, $\Pr = \nu_f \left(\rho \, c_p \right)_f / k_f$ are the Grashof and Prandtl numbers?

To simplify the governing system (6)–(8), we also introduce the following transformations:

$$\left. \begin{aligned} & \eta = \left(\frac{Gr}{x} \right)^{1/4} y, \qquad \psi = Gr^{1/4} \ x^{3/4} f(\eta), \\ & u = \frac{1}{r} \frac{\partial \psi}{\partial y} = \left(x Gr \right)^{1/2} \ f'(\eta) , \\ & v = -\frac{1}{r} \frac{\partial \psi}{\partial x} = \frac{1}{4} \left(\frac{Gr}{x} \right)^{1/4} \left(\eta \ f'(\eta) - f(\eta) \right). \end{aligned} \right\} . \tag{10}$$

Where dashes mean differentiation concerning η, the continuity equation (6) is automatically satisfied. So, the system of equations (7) and (8) becomes:

$$\begin{aligned} & \frac{D_1}{D_2} f''' + \frac{3}{4} f \, f'' - \frac{1}{2} f'^2 + \frac{1}{D_2} \xi \\ & \left\{ 2 f' f''' - 3 f''^2 - \left[\eta - \eta f' + 3 f \right] f'''' \right\} + \frac{D_3}{D_2} \theta = 0, \end{aligned} \tag{10}$$

$$\theta'' + \frac{3}{4} \frac{D_5}{D_4} \Pr f \, \theta' = 0 , \tag{11}$$

The boundary conditions (9) can be also transformed into the following form

$$\left.\begin{array}{llll}\eta=0: & f=f_w, & f'=0, & \theta=1 \\ \eta\to\infty: & f'=0, & f''=0, & \theta=0\end{array}\right\}, \quad (12)$$

Where, $f_w=-\frac{4}{3}v_0(x)\left(Gr/x\right)^{-1/4}$ is the suction parameter ($f_w>0$) and injection parameter ($f_w<0$), and is the viscoelasticity parameter.

3. Numerical procedure

The steady two-dimensional hybrid nanofluids flow over a vertical permeable surface in the presence of an inclined magnetic field and a non-uniform heat source/sink. The shooting method in conjunction with the Runge-Kutta algorithm is used to solve the system of coupled nonlinear ordinary differential equations (10) and (11) with boundary conditions (12), see Fig. 5 for the Mathematical model's flow chart structure(Afify&Elgazery,2012) (Elgazery,2019).

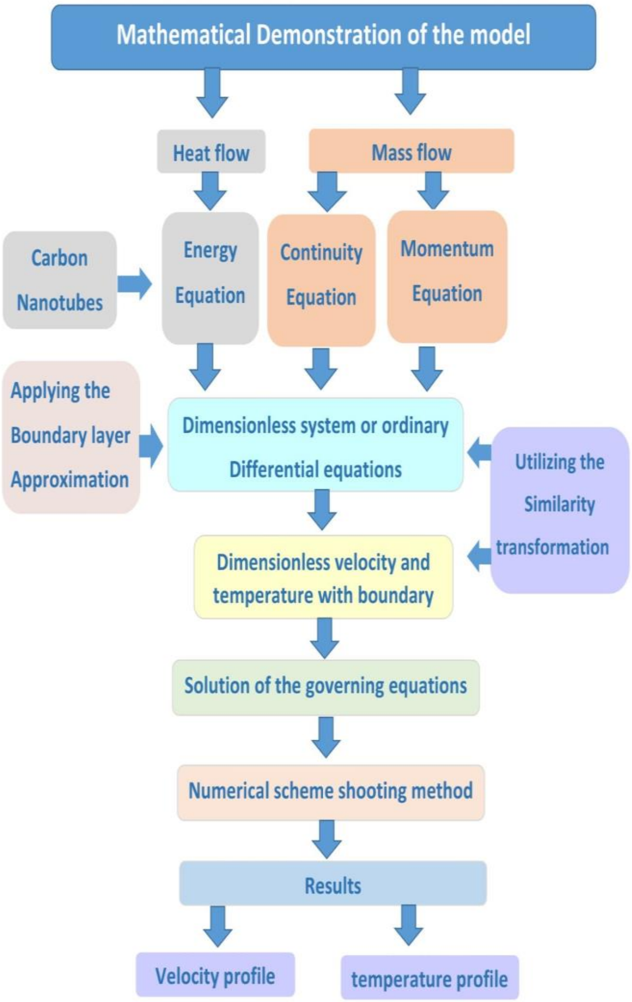

Figure 5. Mathematical model's flow chart structure

4. Discussion

Wastewater typically contains a variety of dissolved and suspended solids, organic matter, and other pollutants, which can increase its viscosity compared to pure water. The equations were solved, and a set of results were obtained, which were displayed in the graphics.

Figure 6 represents the velocity curves in the case of adding SWCNT, and this explains the effect of the concentration of nanoparticles ϕ. The addition of SWCNTs to water can further decrease viscosity, depending on the concentration and dispersion of the nanoparticles. It is observed that the speed of the fluid increased when the viscosity of the fluid was reduced. This means that adding SWCNT to wastewater makes it purer. In other words, there is a process of purifying wastewater happens.

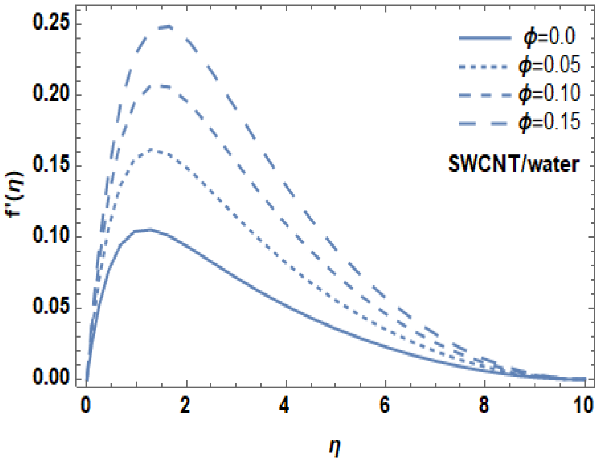

Figure 6. The role of adding SWCNT to wastewater

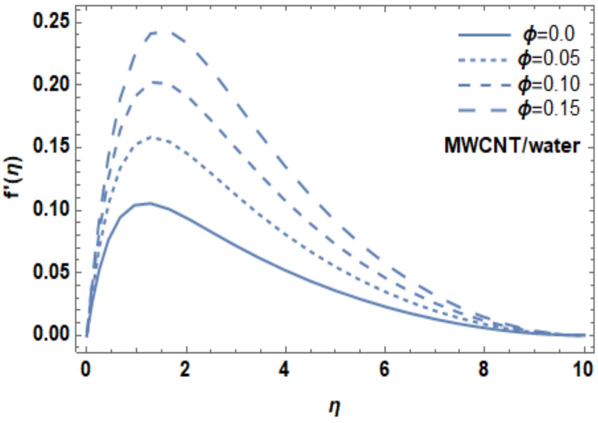

Figure 7. The role of adding MWCNT to wastewater

We will add another type of carbon nano called MWCNT, which is represented in Fig. 7. This figure represents that, at the beginning when the concentration of nanoparticles equals a regular (simple) fluid which means the fluid without adding any type of carbon nanotube that in our research represents wastewater by adding MWCNT, the value of the velocity of the fluid increases. Consequently, when the volume of nanoparticles increases the wastewater velocity will increase compared to the value of $\phi = 0.05$, and so on. Thus, the volume of nanoparticles plays a positive role in the flow rate.

From the last two figures, the addition of carbon nanotubes, such as SWNCT or MWCNT, has an effective and essential role in purifying wastewater. As well as of the nanoparticle concentration also has a vital impact on wastewater treatment.

From the first look in Fig. 8, the impact of nanoparticle concentration on flow rate is seen. This graph shows that the added nanoparticles have a beneficial impact on turbid water's efficiency. Where the turbid water contains various microorganisms and contaminants that may be removed, the wastewater treatment can be also enhanced by mixing two nanomaterials. In other words, the effectiveness of adding a hybrid MWCNT–SWCNT to polluted water is greater than the effectiveness of adding the SWCNT or MWCNT alone, which makes it purer.

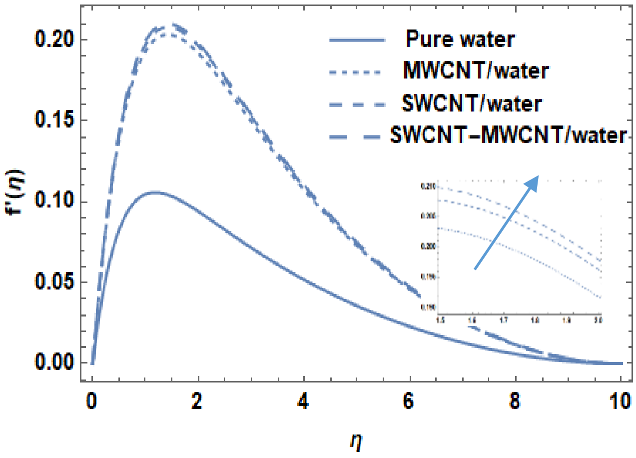

Figure 8. The role of adding hybrid MWCNT-MWCNT to wastewater

5. Conclusion

This research focused on the application of nanotechnology in the purification of wastewater, specifically industrial water. The addition of carbon nanoparticles to the free convective boundary layer viscoelastic fluid flow past a vertical porous cone surface was investigated. The study aimed to provide an affordable and eco-friendly solution for wastewater treatment.

The research employed a numerical approach, transforming the nonlinear partial differential equations governing the problem into a system of ordinary nonlinear differential equations. The shooting method, combined with the Runge-Kutta numerical scheme, was utilized to obtain the numerical solutions.

The key findings of this study contribute to the understanding of wastewater treatment using nanocarbon particles. The investigation of various parameters and variables, such as wastewater, nanocarbon, viscoelasticity, and boundary layer, provided insights into the dynamics of the fluid flow and the purification process. The most important results reached at the end of the research are:

1) adding SWCNT to wastewater makes it purer, the volume of nanoparticles plays a positive role in the flow rate.
2) the addition of carbon nanotubes, such as SWNCT or MWCNT, has an effective and essential role in purifying wastewater. As well as of the nanoparticle concentration also has a vital impact on wastewater treatment .
3) the effectiveness of adding a hybrid MWCNT-SWCNT to polluted water is greater than the effectiveness of adding the SWCNT or MWCNT alone, which makes it purer.

Acknowledgment

It has been a great opportunity to gain a lot of experience in real-time projects, followed by the knowledge of how to design and analyze real projects. For that, we want to thank:

First and foremost we would like to thank God for his never-ending mercy, and grace during the toughest and the greatest times of our lives.

We would like to express our gratitude to our graduation project supervisor Prof. Dr. Nasser S. Elgazery for his effort to teach us, for his support, and his guidance during the long and difficult journey of the project.

We would like to offer our Special thanks to the head of the Mathematics department Prof. Dr.Emad Ali.

We would like to express our deepest gratitude to Masr Elgdeda Language School

for assisting us in implementing the project and utilizing their resources to create the presentation.

We would like to offer our Special thanks to Prof. Dr. Shiemaa M. AbdElall Ibrahim Chemistry Department, Faculty of Education, Ain Shams University for her great effort to help us research this field.

Future Works

We suggest studying using metallic nanoparticles such as Al, Cu, and Ni instead of CNT nanoparticles

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