Bridging the gap: interdisciplinary approaches to mathematical applications

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Abstract

The versatility of trigonometry as a practical analytical tool is particularly evident in forensic science, where it plays a crucial role in crime scene analysis and event reconstruction. This study provides a comprehensive overview of the application of trigonometric principles within various forensic scenarios, from trajectory analysis and source determination to bloodstain pattern interpretation and facial reconstruction from skeletal remains.

Central to crime scene reconstruction is the use of trigonometry to discern the dynamics of a crime through the examination of trajectories, distances, and angles. This paper presents exemplar problems that depict the determination of fall heights and shooter positions, illustrating the method's investigative power. Bloodstain pattern analysis (BPA) further demonstrates trigonometry's forensic utility by using impact angles to reveal critical incident details.

Forensic anthropology benefits from trigonometric analysis for facial reconstruction, ensuring accurate identification. The paper also explores how trigonometry guides the strategic placement of surveillance equipment, a factor increasingly crucial in crime prevention and resolution. In traffic accident forensics, trigonometry aids in estimating vehicle speeds based on skid marks, thereby informing legal outcomes. Lastly, the paper discusses how trigonometric analysis of fingerprint minutiae significantly enhances identification processes.

Targeted at preparatory and high school students, the presented problems aim to illustrate the real-life applications of trigonometry, potentially inspiring interest in mathematics and its use in law enforcement careers. The synthesis of theoretical and applied mathematics underlines the indispensable role trigonometry occupies in modern forensic science, evidencing its vital contribution to public safety and justice.

In conclusion, the paper accentuates trigonometry's substantial impact on forensic science, emphasizing its indispensability in solving crimes and providing an invaluable resource in the pursuit of justice.

Key Words: Trigonometry; Forensic Science; Crime Scene Reconstruction; Bloodstain Pattern Analysis; Surveillance Optimization.
1. **Introduction**: Mathematics is a universal language that underpins much of our daily lives, even though we might not always be aware of it. From the fundamental concepts that shape our understanding of the universe to practical applications in technology, health, finance, and beyond, mathematics is integral to countless aspects of human endeavor. This introduction explores the vast landscape of mathematics' applications, illustrating its indispensable role in both our personal and professional lives.

To incorporate references into the provided text in a professional manner, I will first insert citations at appropriate points within the text, and then list the references at the end, styled similarly to academic or professional articles.

Mathematics is the backbone of technological advancement and engineering. Complex calculations and algorithms enable the design and functioning of computers, smartphones, and the Internet, revolutionizing how we communicate, work, and entertain ourselves [11]. In engineering, principles of calculus, algebra, and geometry are applied to design structures, vehicles, and systems that are safe, efficient, and environmentally sustainable [7]. Mathematical modeling and simulation play critical roles in developing new technologies, from renewable energy sources to advanced medical devices [5].

The exploration of space and our understanding of the physical universe are grounded in mathematical principles. Calculus, for instance, helps astronomers and physicists understand the motion of planets and galaxies, while statistical analysis aids in interpreting data from telescopes and space missions [3]. Mathematics has been essential in landmark achievements, such as landing humans on the Moon and sending probes to the outer planets and beyond [8].

Mathematics has profound applications in health and medicine, from statistical analysis in epidemiology to understand the spread of diseases, to the use of algorithms in genetics for DNA sequencing and in medical imaging to create and interpret scans [9]. Mathematical models are crucial for developing treatment protocols, understanding the dynamics of pandemics, and the creation of drugs and vaccines through pharmacokinetics and pharmacodynamics [13].

The financial world relies heavily on mathematics for modeling markets, assessing risk, and making predictions. Techniques from calculus, statistics, and probability theory help economists understand economic behaviors and cycles, design financial instruments, and manage portfolios [10]. Mathematical models enable the analysis of complex financial systems, guiding policy-making and investment strategies [4].

Mathematics enriches our daily lives in more direct ways than we often realize. Basic arithmetic helps us manage our finances, plan our time, and cook meals. Geometry and algebra help us understand patterns and solve problems in home improvement and arts and crafts. Even our leisure activities, such as playing music or engaging in sports, involve mathematical concepts in tuning instruments or calculating scores and statistics [2].

2. **Mathematical Justice: Exploring the Synergy Between Trigonometry, Forensic Science, and Criminology.**

In the integrated realms of trigonometry, forensic science, and criminology, each discipline significantly contributes to a more profound understanding and investigation of crimes through a multidisciplinary approach. This discussion outlines their interplay and demonstrates their synergy through an exemplary study.

**Trigonometry** plays a crucial role in forensic science, particularly for reconstructing crime scenes involving shootings or blood spatter analysis. The mathematical discipline that investigates the relationships between the angles and sides of triangles is instrumental in determining bullet trajectories or the origins of blood droplets, providing critical insights into the dynamics at a crime scene [6].

**Forensic Science** utilizes scientific principles to validate or challenge evidence in legal contexts. It
encompasses a wide array of techniques, including DNA analysis, fingerprinting, and ballistic analysis, where trigonometry is vital for deciphering the specifics of a crime scene [1].

Criminology is the study of crime, criminal behavior, and the criminal justice system, incorporating insights from psychology, sociology, and forensic science to understand and prevent crime. This field benefits from the detailed reconstructions made possible by applying trigonometric principles in forensic investigations [12].

The application of trigonometry in forensic science provides criminology with sophisticated tools for analyzing the mechanics behind crimes. For example, by examining bullet trajectories, forensic experts can reconstruct shooting incidents, reveal the positions of shooters and victims, identify the possibility of multiple shooters, and elucidate the sequence of events [1]. Such analytical techniques are indispensable for criminological research aimed at understanding criminal behavior and modi operandi.

A study exemplifying the confluence of these disciplines is titled "The Use of Trigonometry in Bloodstain Pattern Analysis to Establish Shooting Positions." This research highlights how applying trigonometric principles to analyze blood spatter patterns can accurately determine a shooter's location in crimes involving firearms. The implications of this study are far-reaching, offering crucial support for courtroom evidence and providing deeper criminological insights into the sequence of crimes [6].

3. Trigonometry in Forensics: Unveiling the Hidden

Trigonometry, a branch of mathematics that studies relationships between side lengths and angles of triangles, plays a pivotal role in the field of forensics and criminology. This intriguing application of trigonometry allows experts to solve mysteries and reconstruct events at crime scenes, making it an essential tool for law enforcement and legal proceedings. Below, we explore how trigonometry is applied in forensics, along with mathematical and word problem examples suitable for students in preparatory schools.

Crime Scene Reconstruction

One of the key uses of trigonometry in forensics is in crime scene reconstruction, particularly in analyzing trajectories and determining the positions of various elements involved in a crime. By examining the angles and distances of bullet paths, blood splatter, or the trajectory of objects, forensic experts can piece together the sequence of events.

Suppose a forensic analyst is trying to determine the height from which a victim fell from a building. The analyst finds that the body landed 15 meters away from the base of the building. Assuming the angle of impact on the ground relative to the point directly below the point of impact is 35 degrees, calculate the height of the building.

Figure (1)

Figure (1) illustrates the scenario where a forensic analyst is determining the height of a fall based on the landing distance and the angle of impact. The building is represented as a vertical line, and the
trajectory of the fall is shown as a dashed red line, ending at the landing point marked with a green circle. This visual representation, combined with trigonometric calculations, clearly demonstrates how the given distance from the base of the building (15 meters) and the angle of impact (35 degrees) are used to estimate the height of the building from which the victim fell.

This problem can be solved using the tangent function in trigonometry, where

\[ \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}. \]

Here, \( \theta \) is the angle of impact, the opposite side is the height of the building, and the adjacent side is the distance from the base (15 meters). Therefore, the height from which the victim fell is approximately 10.5 meters.

**Determining the Source of a Projectile**

Another application of trigonometry in forensics is determining the source of a projectile, such as a bullet. This involves calculating the trajectory path, which can help in identifying the location of the shooter.

A forensic analyst needs to determine the height of a shooter who fired a bullet that struck a wall. The bullet hole was found 1.2 meters above the ground. The shooter was standing 50 meters away from the wall. The bullet was fired at a downward angle of 5 degrees relative to the horizontal. Using this information, calculate the height of the shooter.

To calculate the shooter's height, we employ trigonometric principles, specifically focusing on the tangent function, which relates an angle of a right triangle to the ratio of the opposite side (height difference in this case) to the adjacent side (distance from the shooter to the wall). Then we get

\[
\text{Height Difference} = \tan(\theta) \times \text{Distance to Wall}.
\]

By substituting \( \theta \) with 5 degrees and converting it to radians for calculation, and the Distance to the Wall with 50 meters, we find the height difference due to the bullet's descent.

Finally, since the bullet was fired at a downward angle, the shooter's height is the sum of the bullet hole's height and the calculated height difference.

Through calculation, we determined that the height difference was approximately 4.37 meters, and hence, the shooter's height was approximately 5.57 meters.

**Analyzing Blood Splatter Patterns**

Bloodstain pattern analysis (BPA) is a vital forensic process, where the size, shape, and distribution of bloodstains at a crime scene are studied to understand the events that caused the bloodshed. Trigonometry is used to determine the angles at which blood droplets strike surfaces, which can inform investigators about the positions and movements of the victim and assailant during the crime.
At a crime scene, a blood droplet is found to have hit a wall at a height of 1 meter. The bloodstain is elongated, indicating that the blood hit the wall at an angle. If the length of the bloodstain is 1.8 cm and its width is 0.6 cm, calculate the angle of impact of the blood droplet on the wall. Assume the angle is measured with respect to the surface perpendicular to the wall.

The angle of impact can be calculated using the width-to-length ratio of the bloodstain, which correlates with the sine of the impact angle:

$$\sin(\theta) = \frac{\text{width}}{\text{length}} = \frac{0.6}{1.8}$$

Solving for $\theta$ gives the angle of impact.

### Facial Reconstruction and Identification

In forensic anthropology, trigonometry is used in facial reconstruction from skeletal remains. By analyzing the angles and proportions of the skull, experts can create models that approximate the victim's appearance, which can be instrumental in identification efforts.

Forensic anthropologists utilize the Frankfurt Plane, an imaginary horizontal line that extends from the bottom of the eye sockets to the opening of the ears, to properly align skulls during reconstruction efforts. Given that the vertical distance from the Frankfurt Plane to the top of the skull is 15 cm, and the horizontal distance from the same plane to the most forward point of the face is 10 cm, we are tasked with calculating the angle between these two points. This angle is crucial for reconstructing the facial profile accurately.
horizontal distance of 10 cm from the plane to the face's most forward point.

Employing trigonometric principles, we determined this angle to be approximately 56.31 degrees. This calculation is vital for forensic reconstructions, as it helps in accurately determining facial angles and skull positioning.

Graph (4) illustrates the geometrical relationship underlying our calculations. A triangle is formed with the base along the Frankfurt Plane, extending 10 cm forward to represent the distance to the face's most forward point, and a vertical line extending 15 cm upwards to indicate the distance to the top of the skull. The resulting right-angled triangle allowed us to calculate the desired angle at the intersection of these two lines, using the arctangent function derived from the triangle's dimensions. This visual representation not only substantiates our mathematical approach but also provides a clear visual aid for understanding the spatial relationship critical for forensic facial reconstruction.

**Determining the Position of Surveillance Cameras**

In modern forensic analysis, determining the optimal placement of surveillance cameras can be crucial in both preventing and solving crimes. Trigonometry can be applied to calculate the best angles and heights at which to install these cameras to maximize coverage and ensure critical areas are monitored effectively.

A security team plans to install a surveillance camera atop a 10-meter high pole. The camera is designed with a downward viewing angle of 45 degrees. The objective is to determine the horizontal distance from the base of the pole to the point where the camera's view reaches the ground, ensuring the camera covers a crucial area of interest effectively.

Figure (5) visually represents the surveillance camera setup based on the corrected problem statement. The camera is installed at the top of a 10-meter high pole, with a downward viewing angle of 45 degrees. Due to the 45-degree angle, the horizontal distance from the base of the pole to where the camera's view reaches the ground is equal to the height of the pole, which is 10 meters.

Since the camera's viewing angle is 45 degrees downward, and we're given the height of the pole, we can model this scenario with a right-angled triangle. The height of the pole represents the opposite side of the angle, and the horizontal distance to where the camera's view reaches the ground represents the adjacent side of the angle. In a right-angled triangle with a 45-degree angle, the lengths of the opposite and adjacent sides are equal. Therefore, the horizontal distance from the base of the pole to where the camera's view reaches the ground is equal to the height of the pole, which is 10 meters.

**Estimating the Speed of Vehicles in Traffic Accidents**

Forensic experts often need to estimate the speed of vehicles involved in traffic accidents. Trigonometry can play a vital role in analyzing the angles of impact, skid marks, and the point of collision, which can provide estimations of the vehicles' speed at the time of the accident.

After a traffic accident, skid marks from a car are measured to be 20 meters long. If the friction coefficient between the tire and the road is known
to be 0.7, forensic analysts can use trigonometry and physics to estimate the car's speed at the beginning of the skid. The formula involves the law of conservation of energy and trigonometric functions to calculate speed based on the length of skid marks and the friction coefficient.

To solve this problem, we use physics principles, particularly the work-energy principle. The work done by the friction force as the car skids to a stop is equal to the kinetic energy the car had just before it began to skid. The formula for kinetic energy (KE) is:

\[ KE = \frac{1}{2}mv^2, \]

where \( m \) is the mass of the car (which will actually cancel out in our calculations, so it's not needed), and \( v \) is the velocity of the car at the beginning of the skid.

The work done by friction (\( W \)) is given by:

\[ W = f \cdot d, \]

where \( f \) is the frictional force, and \( d \) is the distance (20 meters in this case). The frictional force can be further defined using the friction coefficient (\( \mu \)) and the normal force (\( N \)), which for a flat road is equal to the weight of the car (\( mg \)), where \( g \) is the acceleration due to gravity (9.81 m/s\(^2\)). Hence, \( f = \mu N = \mu mg \).

Equating the work done by friction to the initial kinetic energy gives us:

\[ \mu mgd = \frac{1}{2}mv^2. \]

From this, we can solve for \( v \), noting that the mass of the car (\( m \)) cancels out:

\[ v = \sqrt{2\mu gd}. \]

Plugging in \( \mu = 0.7 \), \( g = 9.81 \text{ m/s}^2 \), and \( d = 20 \text{ m} \), we can calculate \( v \).

The car's speed at the beginning of the skid was approximately 16.57 meters per second.

To visualize the relationship between the skid distance and the initial speed for various friction coefficients, we can graph the speed as a function of skid distance for a fixed friction coefficient of 0.7. Figure (6) shows how speed increases with the length of the skid mark, given the constant friction coefficient.

### Figure (6)

**Sound Localization in Crime Scene Investigation**

Trigonometry plays a role in sound localization, helping investigators determine the source of a sound (such as a gunshot or explosion) based on recordings from multiple locations.

After a gunshot is heard in an urban area, three different surveillance devices record the sound at slightly different times. By analyzing the time delays and using the known positions of the devices, investigators can triangulate the sound's origin using trigonometry.

To solve and graph this problem, we'll first need to set up a model representing the situation. Suppose we have three surveillance devices located at different positions, each with its own coordinates. When a gunshot is heard, the sound reaches these devices at slightly different times due to their varying distances from the source. The speed of
sound in air (under standard conditions) is approximately 343 meters per second (m/s).

Let's assume the positions of the three devices and the times at which they recorded the gunshot sound. We'll use these to calculate the position of the gunshot's origin. The basic idea is to set up equations based on the distances (which can be derived from the speed of sound and the time delays) and then solve for the location of the gunshot.

(i) Device positions: Let's say we have three devices at coordinates $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$.

(ii) Sound arrival times: Each device records the sound at times $t_1$, $t_2$, and $t_3$, respectively.

(iii) Speed of sound: $v = 343 \text{ m/s}$.

The distances to the gunshot source from devices A, B, and C can be represented as $d_1$, $d_2$, and $d_3$ respectively. These distances can be calculated using the formula $d = v \cdot t$, where $t$ is the time delay (the difference in time from when the sound was emitted to when it was received by each device).

Let's arbitrarily set the positions of the devices and assume time delays to triangulate the gunshot's origin. We'll solve the problem using these assumptions and then graph the positions of the devices and the calculated origin of the gunshot sound.

Figure (7) shows the triangulation of the gunshot's origin based on the sound recordings from three different surveillance devices. The devices are marked in red, and their positions were predefined for this demonstration. The blue marker indicates the calculated origin of the gunshot, which, based on our model and assumptions, is approximately at coordinates (49.98, 33.33).

This calculation uses the known positions of the devices, the speed of sound, and the time delays in the sound reaching each device. By minimizing the difference between the calculated distances (based on time delays and the speed of sound) and the actual distances, we were able to estimate the origin of the gunshot sound. This method demonstrates how triangulation and principles of physics can be applied in practical scenarios, such as forensic investigations in urban environments.

Enhancing Fingerprint Analysis

Trigonometry can enhance the analysis of fingerprints at a crime scene. By calculating the angles and distances between minutiae (unique points in fingerprint patterns), experts can more accurately match fingerprints to individuals, even from partial or smudged prints.

A partial fingerprint is lifted from a crime scene, with three minutiae visible. Forensic experts create a triangle connecting these points and calculate the internal angles using the law of cosines. This geometric information is compared against a database to find a match.

In this investigation, a partial fingerprint was retrieved from a crime scene, presenting three distinct minutiae. These minutiae were geometrically linked to form a triangle, enabling a precise analysis to assist in the identification process. By applying the law of cosines, we calculated the internal angles of the triangle, providing vital geometric data for forensic comparison.
The illustration portrays a triangle formed by the connection of three minutiae, identified as points $A$, $B$, and $C$. This triangle, distinct from the previously examined example, has side lengths of 5, 7, and 9 units reflecting hypothetical distances between the minutiae. The representation aims to visualize the spatial relationship and geometric configuration of the minutiae, crucial for forensic analysis.

Utilizing the law of cosines, the internal angles of the triangle were calculated. The results reveal angles of approximately $33.56^\circ$ at point $A$, $50.70^\circ$ at point $B$, and $95.74^\circ$ at point $C$. These angles contribute to the geometric profile of the fingerprint, enabling a detailed comparison with database entries.

The geometric analysis of fingerprint minutiae through the law of cosines offers a systematic approach to forensic identification. By transforming the minutiae spatial arrangement into a quantifiable geometric model, it facilitates the matching process against a comprehensive fingerprint database. This case study exemplifies the application of geometric principles in forensic science, highlighting the potential for precise identification in criminal investigations.

Through these varied applications, it's evident that trigonometry is not just an abstract mathematical discipline but a practical tool that has real-world applications in forensics and criminology. These problems demonstrate how mathematical knowledge can be applied to solve complex problems, making trigonometry an invaluable resource in the pursuit of justice and public safety. Engaging with such problems not only enhances students' mathematical skills but also opens their eyes to potential career paths that combine their interests in mathematics and law enforcement.

4. Beyond the Numbers: Final Reflections on Trigonometry's Forensic Implications

Mathematics and trigonometry are fundamental components in the toolkit of modern forensic science and criminology. They are the silent workhorses behind the scenes, providing rigorous methodologies for solving crimes and piecing together the puzzle of events that transpire at crime scenes. Trigonometry, in particular, with its focus on the relationships between the sides and angles of triangles, is indispensable for tasks ranging from reconstructing crime scenes and analyzing bullet trajectories, to enhancing the reliability of fingerprint analysis.

The utilization of trigonometry in crime scene investigations illustrates the practical applications of mathematical concepts that are often perceived as abstract. By bringing a level of precision to measurements and enabling the reconstruction of events with mathematical accuracy, trigonometry empowers law enforcement and investigative agencies to draw conclusions that are both reliable and defendable in legal proceedings. This critical application reinforces not only the relevance of mathematics in real-world scenarios but also its role in serving justice and public safety.

Integrating Forensic Trigonometry into Preparatory and High School Curricula

To bridge the gap between theoretical mathematics and practical application, the inclusion of forensic trigonometry in preparatory and high school curricula is recommended. Such integration would not only enrich the learning experience but also illuminate career paths that combine mathematical expertise with public service. Here are some recommendations for
incorporating forensic trigonometry into educational textbooks and materials:

(i) Contextual Learning:

- Develop chapters or sections that outline the use of trigonometry in forensic science with real-world scenarios and case studies.
- Present problems that allow students to step into the role of a forensic analyst, applying mathematical concepts to solve crimes.

(ii) Collaborative Projects:

- Encourage group projects that simulate crime scene investigations, requiring students to use trigonometric functions to derive conclusions from given data.
- Involve local law enforcement agencies to provide expert talks or workshops, reinforcing the curriculum with hands-on learning experiences.

(iii) Technology Integration:

- Leverage computer software and applications that can simulate crime scene reconstructions, providing an interactive platform for students to apply trigonometry in a virtual setting.

(iv) Interdisciplinary Approach:

- Create interdisciplinary units that couple mathematics with subjects like biology (for blood spatter analysis) or physics (for accident reconstruction), emphasizing the interconnected nature of scientific disciplines in forensic applications.

(v) Skill Development:

- Focus on developing critical thinking and problem-solving skills through complex, inquiry-based forensic scenarios.
- Incorporate activities that improve spatial visualization skills, an essential competency for understanding trigonometric principles.

(vi) Career Education:

- Highlight the various careers in forensic science and criminology that utilize mathematics, providing students with information about the educational pathways and skill sets required for these professions.

By integrating these recommendations into the curricula, educators can provide a more engaging and applicable mathematical education. Students will not only be able to appreciate the value of trigonometry in solving concrete problems but will also be better equipped with the skills necessary for future careers in STEM fields, including forensic science. This approach ensures that students recognize the power and purpose of mathematics beyond the classroom, fostering a generation of learners well-versed in applying their mathematical knowledge to promote justice and community welfare.

5. Conclusion

The application of trigonometry in the domain of forensic science exemplifies the profound impact that mathematical disciplines can have on real-world issues, particularly in the realm of law enforcement and criminal justice. Through a diverse array of forensic scenarios—from crime scene reconstruction and projectile sourcing to bloodstain pattern analysis and facial reconstruction—the indispensability of trigonometry emerges not only as a theoretical framework but as a practical instrument in the meticulous work of forensic investigation.

The cases presented underscore the role of trigonometric functions in yielding precise measurements and enabling the deduction of facts from complex scenarios. The sine, cosine, and tangent ratios are not mere abstractions but are transformed into the linchpins of methodologies that foster accuracy and reliability in crime scene analysis. By applying these principles, forensic experts are able to piece together fragmented
evidence, reconstructing events with a degree of certainty that is both scientifically sound and legally defensible.

Furthermore, the integration of forensic trigonometry into educational curricula offers a compelling case for the utility of mathematics in addressing societal needs. By framing mathematical concepts within the context of forensic applications, educators can ignite student interest and demonstrate the tangible benefits of mathematical literacy. The inclusion of real-life case studies provides students with a lens through which to view mathematics as a dynamic and responsive field, one that extends far beyond the confines of academia and into the vital operations of community and public safety.

To this end, the proposed educational strategies aim to enhance student engagement, foster interdisciplinary learning, and cultivate the necessary skill sets for future professionals in forensic science. Such an approach does not merely aim to inform but to inspire, encouraging students to envisage a career that harmonizes the rigors of mathematical reasoning with the noble pursuit of justice. In bridging the gap between abstract mathematical theory and its application, we endorse a vision of mathematics as an indispensable ally in the quest to elucidate truth and uphold the rule of law.

This interconnection of mathematics, forensic science, and education heralds a progressive step towards a more informed and just society. By investing in the next generation of learners and equipping them with the tools to apply trigonometry in forensic contexts, we not only enhance their educational journey but also reinforce the integral role of mathematics in the fabric of societal advancement. As we continue to unveil the multifaceted applications of trigonometry in forensics, we affirm the discipline's standing as an intellectual cornerstone, vital in the pursuit of truth within the judicial process and beyond.
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